

MECHANICAL ENGINEERING SERIES

Pierre Ladevèze
Jean-Pierre Pelle

Mastering
Calculations in
Linear and
Nonlinear
Mechanics



Springer

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Frederick F. Ling
Series Editor

Pierre Ladevèze Jean-Pierre Pelle

Mastering Calculations in Linear and Nonlinear Mechanics

Translated by Theofanis Strouboulis

With 143 Figures



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Pierre Ladevèze
Lab. Mécanique et Technologie
Ecole Normale Supérieure de Cachan,
France
61, avenue du Président Wilson
Cachan Cedex 94235, France
ladeveze@lmt.ens-cachan.fr

Jean-Pierre Pelle
Lab. Mécanique et Technologie
Ecole Normale Supérieure de Cachan, France
61, avenue du Président Wilson
Cachan Cedex 94235, France

Series Editor

Frederick F. Ling
Ernest F. Gloyna Regents Chair in Engineering, Emeritus
Department of Mechanical Engineering
The University of Texas at Austin
Austin, TX 78712-1063, USA
and
William Howard Hart Professor Emeritus
Department of Mechanical Engineering,
Aeronautical Engineering and Mechanics
Rensselaer Polytechnic Institute
Troy, NY 12180-3590, USA

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Frederick F. Ling

Mechanical Engineering Series

Frederick F. Ling
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Mastering calculations in linear and nonlinear mechanics

*A posteriori errors
Adaptive control of parameters*

Pierre Ladevèze & Jean-Pierre Pelle

Translated by
Theofanis Strouboulis

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Introduction

Today more than ever, modeling and simulation are central to a mechanical engineer's activity. Increasingly complex models are being used routinely on a daily basis. This revolution, which has just begun, is the result of the extraordinary progress in computer technology in terms of both hardware and software.

In order to represent a real problem, one does not use just a single model, but a series of models. Starting from a first model, called the reference model, practical or economic considerations, along with the wish to take advantage of certain particular situations, often lead to the introduction of additional simplifying hypotheses, called condensation hypotheses, which result in a new, more manageable model. This, for example, is the case of hypotheses which, starting from a continuous model of a medium subjected to a given environment, lead to a "finite element" model involving parameters such as the size and type of the elements, the number of iterations, the duration of the time increments....

Of course, it is imperative not to alter the reference model completely. Therefore, controlling the additional simplifying hypotheses is an obvious and major issue. This has been a constant preoccupation on the industrial level as well as in research. The new situation is that over the last twenty years truly quantitative tools for assessing the quality of a model compared to another reference model have appeared.

This work deals with the control of the hypotheses leading from a mechanical model, usually coming from continuum mechanics, to a numerical model, i.e. the mastery of the mechanical computation process itself. Particular attention is given to structural analysis which, in this context, is the most advanced domain. The term “structure” designates the material envelope, which can consist of metallic materials, composite materials, biomaterials ... in solid, fluid or gaseous environments. The models being studied are not necessarily linear and high degrees of nonlinearity may be present (plasticity, viscoplasticity, unilateral contact...). The objective of structural analysis is to simulate the behavior of a structure subject to various solicitations (prescribed displacements and forces) numerically; in particular, the aim is to evaluate the state of damage of the structure and compare it with one or several limit states. The final stage consists in optimizing the structural parameters. The practical problems concern the dimensioning, optimization, reliability and even the manufacturing process of the object being designed or built.

The basic problem consists in defining and evaluating a measure of the error due to the discretization performed, in this case, by the finite element method.

Two situations must be dealt with, depending on whether the error is evaluated before or after the finite element calculation has been performed.

Today, for the first situation corresponding to what one calls “a priori” errors, only coarse evaluations are available. The second situation is more favorable: the finite element solution constitutes an additional piece of information. It is in the corresponding field of “a posteriori” error evaluation that the first research works on linear problems were published about twenty years ago.