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Mathematical Methods in Robust Control of Linear Stochastic Systems

Second Edition

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To our wives, Viorica, Elena and Dana
for their love, patience and support.

Preface to the First Edition

This monograph presents a thorough description of the mathematical theory of robust linear stochastic control systems. The interest in this topic is motivated by the variety of random phenomena arising in physical, engineering, biological, and social processes. The study of stochastic systems has a long history, but two distinct classes of such systems drew much attention in the control literature, namely stochastic systems subjected to white noise perturbations and systems with Markovian jumping. At the same time, the remarkable progress in recent decades in the control theory of deterministic dynamic systems strongly influenced the research effort in the stochastic area. Thus, the modem treatments of stochastic systems include optimal control, robust stabilization, and H^2-and H^∞-type results for both stochastic systems corrupted with white noise and systems with jump Markov perturbations.

In this context, there are two main objectives of the present book. The first one is to develop a mathematical theory linear time-varying stochastic systems including both white noise jump Markov perturbations. From the perspective of this generalized theory the stochastic systems subjected only to white noise perturbations or to jump Markov perturbations can be regarded as particular cases. The second objective is to develop analysis and design methods for advanced control problems of linear stochastic systems with white noise and Markovianjumping as linear-quadratic control, robust stabilization, and disturbance attenuation problems. Taking into account the maj or role played by the Riccati equations in these problems, the book presents this type of equation in a general framework. Particular attention is paid to the numerical aspects arising in the control problems of stochastic systems; new numerical algorithms to solve coupled matrix algebraic Riccati equations are also proposed and illustrated by numerical examples.

The book contains seven chapters. Chapter 1 includes some prerequisites conceming measure and probability theory that will be used in subsequent developments in the book. In the second part of this chapter, detailed proofs of some new results, such as the Itô-type formula in a general case covering the classes of stochastic systems with white noise perturbations and Markovian jumping, are given. The Itô-type formula plays a cmcial role in the proofs of the main results of the book.

Chapter 2 is mainly devoted to the exponential stability of linear stochastic systems. It is proved that the exponential stability in the mean square of the considered class of stochastic systems is equivalent with the exponential stability of an appropriate class of deterministic systems over a finite-dimensional Hilbert space. Necessary and sufficient conditions for exponential stability for such deterministic systems are derived in terms of some Lyapunov-type equations. Then necessary and sufficient conditions in terms of Lyapunov functions for mean square exponential stability are obtained. These results represent a generalization of the known conditions conceming the exponential stability of stochastic systems subjected to white noise and Markovian jumping, respectively.

Some stmctural properties such as controllability, stabilizability, observability, and detectability linear stochastic systems subjected to both white noise andjump Markov perturbations are considered in Chapter 3. These properties play a key role in the following chapters of the book.

In Chapter 4 differential and algebraic generalized Riccati-type equations arising in the control problems of stochastic systems are introduced. Our attention tums to the maximal, minimal, and stabilizing solutions of these equations for which necessary and sufficient existence conditions are derived. The final part of this chapter provides an iterative procedure for computing the maximal solution of such equations.

In the fifth chapter of the book, the linear-quadratic problem on the infinite horizon for stochastic systems with both white noise and jump Markov perturbations is considered. The problem refers to a general situation: The considered systems are subjected to both state and control multiplicative white noise and the optimization is performed under the class of nonanticipative stochastic controls. The optimal control is expressed in terms of the stabilizing solution of coupled generalized Riccati equations. As an application of the results deduced in this chapter, we consider the optimal tracking problem.

Chapter 6 contains corresponding versions of some known results from the deterministic case, such as the Bounded Real Lemma, the Small Gain Theorem, and the stability radius, for the considered class of stochastic systems. Such results have been obtained separately in the stochastic framework for systems subjected to white noise and Markov perturbations, respectively. In our book, these results appear as particular situations of a more general class of stochastic systems including both types of perturbations.

In Chapter 7 the γ-attenuation problem of stochastic systems with both white noise and Markovian jumping is considered. Necessary and sufficient conditions for the existence of a stabilizing γ-attenuating controller are obtained in terms of a system of coupled game-theoretic Riccati equations and inequalities. These results allow one to solve various robust stabilization problems of stochastic systems subjected to white noise and Markov perturbations, as illustrated by numerical examples.

The monograph is based entirely on original recent results of the authors; some of these results have been recently published in control joumals and conferences proceedings. There are also some other results that appear for the first time in this book.

This book is not intended to be a textbook or a guide for control designers. We had in mind a rather larger audience, including theoretical and applied mathematicians and research engineers, as well as graduate students in all these fields, and, for some parts of the book, even undergraduate students. Since our intention was to provide a self-contained text, only the first chapter reviews known results and prerequisites used in the rest of the book.

The authors are indebted to Professors Gerhard Freiling and Isaac Yaesh for fruitful discussions on some of the numerical methods and applications presented in the book.

Finally, the authors wish to thank the Springer publishing staff and the reviewer for carefully checking the manuscript and for valuable suggestions.

Preface

This new edition has nine chapters and it includes some new developments and results in robust control of linear stochastic systems.

In Chapter 1 properties of homogeneous Markov processes with countable infinite number of states are given together with Itô–type formula for stochastic systems with white noise perturbations and infinite Markov jumping. Lebesgue's Theorem and Fatou's Lemma for discrete measures are also presented.

Chapter 2 is new. The properties of the Minkovski norm are presented. The main purpose is to provide a characterization for the exponential stability of the linear differential equations with positive evolution on ordered Banach spaces. This characterization is given in terms of the existence of some global defined and bounded solutions of some suitable forward or backward affine differential equations and in terms of some forward or backward affine differential inequalities.

The problem of robustness of exponential stability with respect to some additive perturbations modeled by positive operator valued functions is analized in the case when the involved operators are periodic. The last part of the chapter is devoted to the investigation of the properties of linear evolution operators associated to Lyapunov type differential equations on the Banach spaces $\mathcal{S}_n^d$, $\mathcal{S}_n^\infty$ and $\ell^1(\mathbf{Z}_+, \mathcal{S}_n)$. Criteria for exponential stability of the Lyapunov type differential equations on $\mathcal{S}_n^\infty$ and $\mathcal{S}_n^d$ (the latest being also presented in the second chapter of the first edition) are finally derived as direct consequences of the criteria obtained in the general case.

The novelty of Chapter 3 is the characterization of exponential stability in mean square for stochastic linear differential equations perturbed both by multiplicative white noise and by an infinite Markov process. This is based on the representation theorem of the anticausal linear evolution operator defined by a Lyapunov type differential equation on the space $\mathcal{S}_n^\infty$ and on the criteria of exponential stability of the corresponding linear differential equation presented in Chapter 2.

Chapter 4 is the third chapter of the first edition.

Most of the fifth chapter is new. Bounded global solutions for a wide class of nonlinear differential equations (called GRDE-Generalized Riccati Differential Equations) on an ordered Banach space of symmetric matrices are analyzed. They include as particular cases the systems of Riccati–type equations arising in the stochastic linear control (SGRDE Stochastic Generalized Riccati Differential Equations). Comparison theorems and necessary and sufficient conditions for bounded maximal, minimal and stabilizing solutions of the GRDE and then by consequence, the corresponding results for the SGRDE which are also included in Chapter 4 of the first edition, are provided.

For the sake of clarity, the fifth chapter of the first edition has been split in two chapters, namely Chapter 6 and Chapter 7 of this edition. A new section treating a Kalman filtering problem for stochastic systems with state–dependent noise and Markovian jumping has been included in Chapter 7.

Chapters 8 and 9 are just the sixth and the seventh chapters of the first edition. In the final part of Chapter 9, a new section presenting a mixed H_2/H_∞ filtering problem has been introduced.

The authors wish to thank to Professors G. Freiling, T. Damm, I. Yaesh, O.L.V. Costa, M.D. Fragoso and V. Ungureanu for fruitful discussions on some general properties of differential equations with positive evolution on ordered Banach spaces, numerical methods and applications presented in the book.

We also should like express our gratitude to Mrs. Viorica Dragan and Mr. Catalin Dragan for their hard work in typing the manuscript.

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