VECTOR OPTIMIZATION

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Adaptive Scalarization Methods in Multiobjective Optimization

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Vector Optimization

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To my family,
Paul, Sabine, Susanne, and Claudia,
and especially to Tom.

Preface

In many areas in engineering, economics and science new developments are only possible by the application of modern optimization methods. The optimization problems arising nowadays in applications are mostly multiobjective, i. e. many competing objectives are aspired all at once. These optimization problems with a vector-valued objective function have in opposition to scalar-valued problems generally not only one minimal solution but the solution set is very large. Thus the development of efficient numerical methods for special classes of multiobjective optimization problems is, due to the complexity of the solution set, of special interest. This relevance is pointed out in many recent publications in application areas such as medicine ([63, 118, 100, 143]), engineering ([112, 126, 133, 211, 224], references in [81]), environmental decision making ([137, 227]) or economics ([57, 65, 217, 234]).

Considering multiobjective optimization problems demands first the definition of minimality for such problems. A first minimality notion traces back to Edgeworth [59], 1881, and Pareto [180], 1896, using the natural ordering in the image space. A first mathematical consideration of this topic was done by Kuhn and Tucker [144] in 1951. Since that time multiobjective optimization became an active research field. Several books and survey papers have been published giving introductions to this topic, for instance [28, 60, 66, 76, 112, 124, 165, 188, 189, 190, 215]. In the last decades the main focus was on the development of interactive methods for determining one single solution in an iterative process. Thereby numerical calculations alternate with subjective decisions of the decision maker (d. m.) till a satisfying solution is found. For a survey of interactive methods see [28, 124, 165].

Based on an extreme increase in computer performances it is now possible to determine the entire efficient set. Having an approximation of the whole solution set available the decision maker gets a useful insight in the problem structure and important information are delivered like trade-off information. Thereby trade-off is the information how the improvement of one objective function leads to a deterioration of the other objectives. The importance of approximating the complete efficient set is thus also emphasized in many applications. Especially in engineering it is interesting to know all design alternatives ([119]). Hence nowadays there is an increasing interest in methods for approximating the whole solution set as also the high number of papers related to this topic demonstrates, see for instance [10, 40, 82, 81, 84, 83, 106, 139, 164, 182, 196, 197].

For the determination of approximations of the efficient set several approaches have been developed, as for example evolutionary algorithms (for surveys see [31, 41, 112, 228, 246]) or stochastic methods ([194]). A large class of methods is based on scalarizations. This means the replacement of the multiobjective optimization problem by a suitable scalar optimization problem involving possibly some parameters or additional constraints. Examples for such scalarizations are the weighted sum ([245]) or the ε-constraint problem ([98, 159]). In this book we concentrate on the scalarization approach and we set especially value on the scalar problem according to Pascoletti and Serafini ([181]). However, many other existing auxiliary problems, which will also be presented, can be related to that method.

As generally not the entire efficient set can be computed an approximation is instead generated by solving the scalar problems for various parameters. The information delivered to the decision maker by such an approximation depends mainly on the quality of the approximation. Too many points are related to a high computational effort. Too few points means that some parts of the efficient set are neglected. Hence it is important to take quality criteria as discussed for instance in [32, 43, 101, 141, 191] into account. An approximation with a high quality is given if it is stinted but also representative, i. e. if the approximation points are spread evenly over the efficient set with almost equal distances.

We develop in this book methods for generating such approximations for nonlinear differentiable problems. For these methods the sensitivity of the scalar problems on their parameters are examined. These sensitivity results are used for developing an adaptive parameter control. Then, without any interaction from the decision maker, the choice of the parameters is in such a way controlled during the procedure, that the generated approximation points have almost equal distances.

Thereby we consider very general multiobjective problems and allow arbitrary partial orderings induced by a closed pointed convex cone in the image space (like in [81, 96, 106, 181, 230]) using the notion of K-minimality as given in [14, 102, 122, 124, 190, 243]. The partial ordering of the Edgeworth-Pareto-minimality concept represented by the natural ordering cone, the positive orthant, is included as a special case. More general orderings rise the applicability of our methods as the decision makers get more freedom in the formulation of the optimization problems. Preference structures can be incorporated, which cannot be expressed explicitly by an objective function (see Example 1.5 and [230, Example 4.1]). In decision theory in economics it is a well-known tool to use arbitrary partial orderings for modeling the relative importance of several criteria for a d. m. as well for handling groups of decision makers ([235]).

For example in [116, 117] convex polyhedral cones are used for modeling the preferences of a d. m. based on trade-off information facilitating multi-criteria decision making. In portfolio optimization ([5]) polyhedral cones in $\mathbb{R}^m$ generated by more than m vectors, as well as non-finitely generated cones as the ice-cream cone, are considered. Besides, orderings, other than the natural ordering, are important in [85] where a scalar bilevel optimization problem is reformulated as a multiobjective problem. There a non-convex cone which is the union of two convex cones is used. Helbig constructs in [106] cone-variations as a tool for finding EP-minimal points, see also [134, 237]. In addition to that Wu considers in [238] convex cones for a solution concept in fuzzy multiobjective optimization. Hence, multiobjective optimization problems w. r. t. arbitrary partial orderings are essential in decision making and are further an important tool in other areas. Therefore we develop our results w. r. t. general partial orderings.

This book consists of three parts. In the first part theoretical basics of multiobjective optimization are introduced as for instance minimality notions and properties of ordering cones especially of polyhedral cones. Scalarizations are discussed with a special focus on the Pascoletti-Serafini scalarization. Further, sensitivity results for these parameter depended scalar problems are developed like the first order derivative information of the local minimal value function.

The second part is devoted to numerical methods and their application. Quality criteria for approximations of the efficient set are introduced and the main topic of this book, the adaptive parameter control using the sensitivity results developed before, is constructed. We differentiate thereby between the treatment of biobjective optimization problems and general multiobjective optimization problems. The gained algorithms are applied to various test problems and to an actual application in intensity modulated radiotherapy.

The book concludes in the third part with the examination of multiobjective bilevel problems and a solution method for those kinds of problems, which is also applied to a test problem and to an application in medical engineering.

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Erlangen, January 2008 *Gabriele Eichfelder*

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Part I

Theory